RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

SECOND YEAR [2016-19]
B.A./B.Sc. THIRD SEMESTER (July – December) 2017
Mid-Semester Examination, September 2017

Date : 12/09/2017 **MATHEMATICS (Honours)**

Time : 11 am – 1 pm Paper : III Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

(*Answer **any two** questions*) [2×5]

1. A square PQRS of diagonal 2a, is folded along the diagonal PR so that the planes SPR and QPR are at right angles. Show that the shortest distance between SR and PQ is then $\frac{2a}{\sqrt{3}}$.

2. Find the smallest sphere which touches the line $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z-6}{1}$ and $\frac{x+3}{7}=\frac{y+3}{-6}=\frac{z+3}{1}$.

3. If a right circular cone of the semi-vertical angle θ passes through the x and y-axes and also through the line x = y = z, show that $\sec^2\theta = 9-4\sqrt{3}$.

Group – B

(*Answer **any two** questions*) [2×7·5]

4. A particle moves in one plane under a force which is always perpendicular and towards a fixed straight line on the plane, its magnitude being $\mu \div$(distance from the line)^2. If initially it be at a distance 2a from the line and be projected with a velocity $\sqrt{\frac{\mu}{a}}$ parallel to the line, prove that the path traced out by it is a cycloid.

5. A particle hangs at rest at the end of an elastic string whose unstretched length is a. In the position of equilibrium, the length of the string is b and $\frac{2\pi}{n}$ is the time of an oscillation about this position. At time zero when the particle is in equilibrium, the point of suspension begins to move so that its downward displacement is c sin(pt). Show that the length of the string at time t is

$$b-\frac{cnp}{n^2-p^2}\sin(nt)+\frac{cp^2}{n^2-p^2}\sin(pt)\,(p\neq n).$$

6. A particle is projected vertically upwards with a velocity u and the resistance of the air produces a retardation Kv^2, where v is the velocity. Show that the velocity u_1 with which the particle will return to the point of projection is given by $\frac{1}{u_1^2}=\frac{1}{u^2}+\frac{K}{g}$.

Group – C

(*Answer **any five** questions*) [5×5]

7. Let F be a field and V be the space of polynomial functions f from F into F, given by $f(x)=c_0+c_1x+\ldots+c_Kx^K$ for $c_i\in F$, for i = 1, 2, ..., K.

Let us define $(Df)(x) = c_1 + 2c_2x + ... + Kc_Kx^{K-1}$ for any $f \in V$. Prove that D is a linear transformation.

8. Let $T: V \to W$ be a linear transformation where V, W are vector spaces over the field F and V is finite dimensional. Prove that rank (T) + nullity (T) = dim V.

9. V be a finite dimensional vector space over the field F and T be a linear operator on V. Suppose rank (T^2) = rank (T). Prove that the range space and null space of T have only zero vector in common.

10. Let A be an m×n matrix over the field F, $T: F^{n\times1} \to F^{m\times1}$ defined by T(X) = AX. Show that if $m < n$ it my happen that T is onto without being nonsingular. Similarly, show that if m > n we may have T non singular but not onto.

11. Let V be a vector space over the field F and W be a subspace of V. Prove that $\dim\left(\frac{V}{W}\right) = \dim V - \dim W$.

12. Let $V = \mathbb{R}^4$. W be a subspace of V generated by {(1,2,3,4), (2,3,7,9)}. Find two different complement for W in V.

13. Find a basis for the row space of the following matrix $\begin{bmatrix} 1 & 3 & 2 & 1 & 3 \\ 2 & 2 & 1 & 3 & 4 \\ 3 & 5 & 3 & 2 & 2 \\ 4 & 9 & 1 & 5 & 6 \\ 5 & 10 & 5 & 1 & 8 \end{bmatrix}$.

14. For what values of K the planes x + y + z = 2, 3x + y – 2z = K and 2x + 4y + 7z = K + 1; form a triangular prism?

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